MODELLING HEAT TRANSFER IN COOKED TUNA DURING COOLING

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ABSTRACT

The effect of structure and shape on temperature profile of food substances was studied. Three different computer models based on analytical solutions were developed to predict temperature distribution in food. Models 1, 2 and 3 represent a single infinite cylinder, a 2-layer composite infinite cylinder, and a single finite cylinder, respectively. Tuna was used to validate the model and it was found that the calculated temperatures at different locations from model 1 were in good agreement with those from measurements. For the effect of shape on cylindrical shaped food substances, it was found that selecting a proper model depends on the ratio of diameter to length (D/L) and the position distance from the center (x/x_0) of the cylinder. For the effect of structure of materials in the case of 2-layer composite materials which contain different physical and thermal properties in each layer, it was found that the effect of structure of material on the heat transfer depends on the ratio of inner diameter to outer diameter of the cylinder (r_b/r_o).

1. INTRODUCTION

Most biomaterials used in food industry are materials which have irregular shape and nonhomogeneous structure. The effect of shape and structure of food is very important to heat transfer phenomena and causes difficulty in identifying exact condition required during food processing, for example, heating and cooling. It is, therefore, necessary to understand heat transfer behavior due to all these factors. One approach to find the temperature distribution would be an application of analytical method. However, manual method of analytical solutions is time-consuming due to the complexity of mathematical equations. The computer models were then developed based on the analytical solutions.

Most studies are focused on temperature distribution in homogeneous cylindrical shaped food. Perez-Martin, et.al. (1989) developed a simple semi-empirical model to predict the optimal precooking time of albacore canning factories. The fish was assumed to be a homogeneous finite cylinder. The validation was found to be in good agreement with the experimental results. Chau and Snyder (1988) presented a mathematical model for heat transfer in thermally processed shrimp during both the heating phase and cooling phase. The validation shows that the predicted results and the experimental results are in good agreement. This paper has considered 2-layer composite cylindrical shaped food substances having different physical and thermal properties in each layer.

The effect of shape and structure of cylindrical shaped food on heat transfer was studied through models 1, 2 and 3. Model 1 considered cylindrical shaped food as a single infinite cylinder. Model 2 considered cylindrical shaped food as a 2-layer composite infinite cylinder. Model 3 considered cylindrical shaped food as a single finite cylinder.

In this research, tuna was selected as an example of cylindrical shaped food. Tuna is one of the most popular fish in food industry. The variation in size and composition of tuna is due to seasonality. This variation causes difficulty in tuna processing because the amount of time for each step, for example, thawing, steaming, or cooling, varies accordingly. Especially, during cooling, when the cooling for cooked tuna is too long, the problems arise. These problems are, for example, the outer surface of tuna is too dry, the flesh and skin of tuna become more sticky, and the fatty smell occurs due to fat oxidation.

Therefore, the accurate prediction of temperature distribution in tuna during cooling is very important. Knowing geometry, composition, thermal properties, and boundary and initial conditions, mathematical models can be developed based on analytical solutions and used as a tool to predict temperature distribution in tuna. The mathematical models then can be applied to identify and improve the cooling process efficiently.

2. MATERIALS AND METHODS — MODEL FOR TEMPERATURE PREDICTION

Model development

Models 1, 2 and 3 considered cylindrical shaped food as a single infinite cylinder, a 2-layer composite infinite cylinder, and a single finite cylinder, respectively. The development of models 1, 2 and 3 was based upon the following assumptions:

- (a) The problem is transient.
- (b) Materials are considered homogeneous and isotropic.
- (c) Physical and thermal properties of the materials do not vary with temperature.
- (d) The convective heat transfer coefficient (h) and surrounding air temperature (t_a) are uniform and constant.
- (e) Initial temperature is uniform.

Analytical solutions

2.1 Model 1

Figure 1 illustrates model 1 as a single infinite cylinder.

The governing equation for one-dimensional transient heat conduction in an infinite cylinder made of a homogeneous and isotropic material without heat generation is given as the following (Incropera and DeWitt, 1990).

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} = \frac{1}{\alpha} \frac{\partial t}{\partial \theta} \quad (1)$$

subject to boundary conditions:

$$\frac{\partial t}{\partial r} = 0, \quad r = 0 \quad (2)$$

$$\frac{\partial t}{\partial r} + \frac{h}{k}(t - t_a) = 0, \quad r = r_o \quad (3)$$

subject to initial condition:

$$t = t_i, \quad \theta = 0 \quad (4)$$

The analytical solution of model 1 is as follows.

$$t(r, \theta) = 2Bi(t_i - t_a) \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r / r_o) e^{-\lambda_n^2 F_0}}{J_0(\lambda_n) [\lambda_n^2 + Bi^2]} + t_a \quad (5)$$

$$\frac{t(r, \theta) - t_a}{t_i - t_a} = 2Bi \sum_{n=1}^{\infty} \frac{J_0\left(\lambda_n \frac{r}{r_o}\right)}{J_0(\lambda_n) [\lambda_n^2 + Bi^2]} e^{-\lambda_n^2 F_0} \quad (6)$$

Figure 1: A single infinite cylinder under specified condition.

Figure 2: A 2-layer composite infinite cylinder under specified condition.

2.2 Model 2

Figure 2 illustrates model 2 as a 2-layer composite infinite cylinder.

The governing equation for one-dimensional transient heat conduction in a two-region concentric solid cylinder, with perfect thermal contact at the interface and solved by Tittle and Robinson, 1965 (cited by Ozisik, 1968) is as follows.

$$\frac{\partial^2 t_1}{\partial r^2} + \frac{1}{r} \frac{\partial t_1}{\partial r} = \frac{1}{\alpha_1} \frac{\partial t_1}{\partial \theta} \quad , \quad 0 \leq r \leq r_b \quad (7)$$

$$\frac{\partial^2 t_2}{\partial r^2} + \frac{1}{r} \frac{\partial t_2}{\partial r} = \frac{1}{\alpha_2} \frac{\partial t_2}{\partial \theta} \quad , \quad r_b \leq r \leq r_0 \quad (8)$$

subject to boundary conditions:

$$\frac{\partial t}{\partial r} = 0, \quad r = 0 \quad (9)$$

$$k_1 \frac{\partial t_1}{\partial r} = k_2 \frac{\partial t_2}{\partial r} \quad , \quad r = r_b \quad (10)$$

$$k_2 \frac{\partial t_2}{\partial r} + h(t_2 - t_a) = 0 \quad , \quad r = r_0 \quad (11)$$

subject to initial conditions :

$$t_1 = t_i \quad , \quad \theta = 0 \quad (12)$$

$$t_2 = t_i \quad , \quad \theta = 0 \quad (13)$$

The analytical solution of model 2 is as shown below.

$$t_j(r, \theta) = \sum_{n=1}^{\infty} A_n X_{jn}(r) e^{-\alpha_j \beta_{jn}^2 \theta} \quad , \quad j = 1, 2 \quad (14)$$

The details of equation (14) are available in Ozisik, 1968.

2.3 Model 3

Fig. 3 illustrates model 3 as a single finite cylinder.

The governing equation for one-dimensional transient heat conduction in an infinite slab is the following (Ozisik, 1993).

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \frac{\partial t}{\partial \theta} \quad (15)$$

subject to boundary conditions:

$$\frac{\partial t}{\partial x} = 0, \quad x = 0 \quad (16)$$

$$\frac{\partial t}{\partial x} + \frac{h}{k}(t - t_a) = 0, \quad x = x_0 \quad (17)$$

subject to initial condition:

$$t = t_i, \quad \theta = 0 \quad (18)$$

The analytical solution of an infinite slab is shown below.

$$t(x, \theta) = 2(t_i - t_a) \sum_{m=1}^{\infty} \frac{1}{[\beta_m + \sin \beta_m \cos \beta_m]} \sin \beta_m \cos \left(\beta_m \frac{x}{x_0} \right) e^{-\alpha \beta_m^2 \theta} + t_a \quad (19)$$

$$\frac{t - t_a}{t_i - t_a} = 2 \sum_{m=1}^{\infty} \frac{1}{[\beta_m + \sin \beta_m \cos \beta_m]} \sin \beta_m \cos \left(\beta_m \frac{x}{x_0} \right) e^{-\alpha \beta_m^2 \theta} \quad (20)$$

Figure 3: A single finite cylinder under specified condition.

Myers, 1971 (cited by Singh and Heldman 1993) proved mathematically that the temperature ratio of the finite cylinder can be calculated from the multiplication of the temperature ratio of an infinite cylinder and an infinite slab as the following.

$$\left[\frac{t - t_a}{t_i - t_a} \right]_{\text{finite cylinder}} = \left[\frac{t - t_a}{t_i - t_a} \right]_{\text{infinite cylinder}} \times \left[\frac{t - t_a}{t_i - t_a} \right]_{\text{infinite slab}} \quad (21)$$

Substituting equations (6) and (20) into equation (21), the solution is as shown below.

$$\left[\frac{t - t_a}{t_i - t_a} \right]_{\text{finite cylinder}} = D_n D_m e^{-\left[\frac{\beta_m^2}{x_0^2} + \frac{\lambda_n^2}{r_0^2} \right] \alpha \theta} \quad (22)$$

where

$$D_n = 2Bi \sum_{n=1}^{\infty} \frac{J_0 \left(\lambda_n \frac{r}{r_0} \right)}{J_0(\lambda_n) [\lambda_n^2 + Bi^2]}$$

$$D_m = 2 \sum_{m=1}^{\infty} \frac{1}{[\beta_m + \sin \beta_m \cos \beta_m]} \sin \beta_m \cos \left(\beta_m \frac{x}{x_0} \right)$$

Computer program

The computer code was developed for all three models using MATLAB4.00 SIMULINK 1.2c. Each computer model was based on analytical solution. The computer models allow fast calculations on a microcomputer to predict temperature distribution. Input parameters used for models 1, 2 and 3 are shown in Table 1.

3. EXPERIMENTS FOR MODEL VALIDATION

3.1 Raw material and instrument

- Tuna (*Euthynnus affinis*)
- Steam cooker (model No. 1941x, All American)
- Electronic balance (model RC 250s, Startorius)
- Environmental chamber
- Thermocouple type T (copper-constantan) model TT-T-24 (Omega Engineering, Inc., Stamford, CT)
- Vernier caliper
- Data logger (model HR 1300, Yokogawa)
- Walk-in refrigerator
- Anemometer (model DA-42, Digicon)

Figure 4: Positions of thermocouples used in the experiment.

3.2.2 Temperature measurement during cooling of cooked whole tuna

The geometry of cooked whole tuna obtained from preparation of tuna was measured. All thermocouples were calibrated and placed at different locations. The position of each thermocouple is shown in Figure 4. One thermocouple was also placed outside the tuna to measure surrounding air temperatures. The tuna with thermocouples connected to the locations of interest was placed in the environmental chamber until the temperatures were uniform. The tuna was then transferred to a refrigerator at 4°C. The data logger was programmed to take readings at 2-minute intervals. The experiment was repeated.

4. RESULTS AND DISCUSSION

4.1 Model 1 Validation

The computer model 1 was validated using the observation through experiments. The calculated temperatures from model 1 were compared to measurements. The difference between the observed and predicted temperatures was calculated in term of the percent maximum difference in temperature as the following.

$$\% \Delta T_{\max} = \frac{|t_{\exp} - t_{\text{cal}}|_{\max}}{t_i - t_a} \times 100 \quad (23)$$

The observed temperatures at different locations of tuna with initial temperature 60°C were plotted against the corresponding predicted temperature in Figure 5 through 10. The maximum differences between the predicted and observed temperatures at locations 1, 2, 3, 4, 5 and 6 were approximately 2, 15, 4, 13, 2 and 2%, respectively. The maximum difference at location 2, the surface, is slightly high because the surrounding air temperature and the convective heat transfer coefficient used in the model are assumed to be uniform and constant. In addition, the maximum difference at location 4, near the tail, is slightly high because the tail part of tuna is not close to cylinder.

4.2 Effect of structure

As mentioned earlier, model 2 considered cylindrical shaped food as a 2-layer composite infinite cylinder having different physical and thermal properties in different layer but same physical and thermal properties in the same layer. The effect of structure, i.e., the ratio of thermal diffusivity along with the ratio of radius of the inner material to those of the outer material, was investigated by comparing the predicted temperatures from models 1 and 2. The maximum difference in temperature ($\% \Delta T_{\max}$) was calculated using equation (23).

Figure 5: Observed and predicted temperatures at location 1 (center).

Figure 6: Observed and predicted temperatures at location 2 (surface).

Figure 7: Observed and predicted temperatures at location 3 (near the head).

Figure 8: Observed and predicted temperatures at location 4 (near the tail).

Figure 9: Observed and predicted temperatures at location 5 (middle point of the upper part towards the head).

Figure 10: Observed and predicted temperatures at location 6 (middle point of the upper part towards the tail).

Figure 11: The maximum difference in temperature predicted from model 1 and model 2 at the middle point between the inner and outer cylinders.

Figure12: The maximum difference in temperature predicted from model 1 and model 3.

Figure 11 shows that when the ratio of thermal diffusivity of the inner material to that of the outer material (α_i / α_o) increased from 1.6 to 3.0, the maximum difference in temperature between predicted temperatures from models 1 and 2 at different r_b / r_o (7.80%, 15.60%, 31.30%, and 46.90%) increased. At α_i / α_o equals 1.6, the maximum difference in temperature between models 1 and 2 was within 1°C at r_b / r_o less than 31.30%. At α_i / α_o equals 3.0, the maximum difference in temperature between models 1 and 2 was within 1°C at r_b / r_o less than 15.60%.

4.3 Effect of Shape

The effect of shape, i.e., the ratio of diameter to length of the homogeneous cylinder was investigated by comparing the predicted temperatures from models 1 and 3. The maximum difference in temperature ($\% \Delta T_{\max}$) was calculated using equation (23).

The maximum difference in temperature of the predicted temperatures from models 1 and 3 at different ratio of diameter to length (D/L) and different ratio of the axial distance from the center to the half length of food (x / x_o) is given in Figure 12. At D/L is less than 1/4, the maximum difference in temperature between models 1 and 3 was within 1°C at x / x_o less than 0.6.

5. CONCLUSIONS

- (1) Three heat transfer models, based on analytical solutions, were developed to predict temperature distribution in food considering the effect of shape and structure. Models 1, 2 and 3 represent a single infinite cylinder, 2-layer composite infinite cylinder, and a single finite cylinder, respectively.
- (2) The temperature predictions from model 1 were in good agreement with the measurement, especially at location 1 (center).
- (3) At α_i / α_o equals 1.6, the maximum difference in temperature between models 1 and 2 was within 1°C at r_b / r_o less than 31.30%. At α_i / α_o equals 3.0, the maximum temperature difference between models 1 and 2 was within 1°C at r_b / r_o less than 15.60%. Because α_i / α_o and r_b / r_o of tuna were approximately 1.6 and 7.8%, respectively, model 1 was found to be a suitable model.
- (4) The maximum difference in temperature predicted from models 1 and 3 was within 1°C when ratio of diameter to length (D/L) is less than 1/4 and ratio of the axial distance from the center to the half length of food (x / x_o) is less than 0.6. Since D/L of tuna was approximately 1/4, model 1 was recommended.

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7. NOTATION

Bi	Biot number , $\frac{hr_o}{k}$ or $\frac{hx_o}{k}$
C_p	specific heat (J/kg.°C)
D	diameter of cylinder (m)
D_{fi}	thermal diffusivity of the inner cylinder (m ² /s)
D_{fo}	thermal diffusivity of the outer cylinder (m ² /s)
F_o	Fourier number , $\frac{\alpha\theta}{r^2}$
h	convective heat transfer coefficient (W/m ² .°C)
J_o	zero order Bessel function of the first kind
J_1	first order Bessel function of the first kind
k	thermal conductivity (W/m ² .°C)
L	finite cylinder length (m)
m	order of series
n	order of series
N	numbers of data points
p	order of Bessel function
r	position variable in radial direction (m)
r^*	dimensionless distance from center along radius
r_b	radius of inner cylinder (m)
r_o	radius of outer cylinder (m)
t	temperature (°C)
t_a	constant temperature of surrounding air (°C)
t_{exp}	observed temperature (°C)
t_{cal}	predicted temperature (°C)
t_i	initial temperature (°C)
t_s	surface temperature (m)
v	air velocity (m/s)
x	position variable in axial direction (m)
x_o	half-length of cylinder (m)
Y_o	zero order Bessel function of the second kind
Y_1	first order Bessel function of the second kind

Greek letters

β_m	Roots of a transcendental equation
λ_n	Roots of a transcendental equation
α	thermal diffusivity (m ² /s)
α_i	thermal diffusivity of the inner cylinder (m ² /s) = D_{fi}
α_o	thermal diffusivity of the outer cylinder (m ² /s) = D_{fo}
θ	time (s)
ρ	density (kg/m ³)

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