DESIGN OF ROBUST H_∞ POWER SYSTEM STABILIZER USING NORMALIZED COPRIME FACTORIZATION

I. Ngamroo* and S. Dechanupaprittha

Electrical Power Engineering Program
Sirindhorn International Institute of Technology
Thammasat University, Pathumthani, 12121 Thailand

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ABSTRACT

This paper proposes a new design procedure of robust power system stabilizers (PSS) using H_∞ control via normalized coprime factorization (NCF) approach. The design procedure of the proposed PSS is systematically described. Moreover, the selection method of the weighting function in H_∞ control design is explained in a simple manner. The performance and robustness of the proposed PSS are investigated in comparison with the conventional PSS by examining the case of a single machine connected to an infinite bus (SMIB) system. The simulation results are illustrated to ensure the effectiveness of the proposed PSS.

Keywords : *Power system stabilizer, H_∞ control theory, normalized coprime factorization, robust control.*

1. INTRODUCTION

At present, power system stabilizers (PSSs) are widely used to alleviate the problem of low-frequency oscillations in interconnected power systems. As an auxiliary controller, PSS provides a supplementary signal at the voltage reference input of the automatic voltage regulator (AVR) to enhance the dynamic stability of a system^{1,2}. Over the years, several methods were proposed for designing PSSs^{3,4,5}. Nevertheless, the robustness of these PSSs against system uncertainties is not satisfactory. To overcome this problem, the robust control theory was applied to improve overall performance of PSSs.

The H_∞ control theory is one of an excellent robust control theory for designing PSSs. The designed H_∞ PSS via mixed sensitivity approach reveal the high performance and robustness^{3,4}. However, the weighting function in H_∞ control design cannot be simply obtained, due to the trade-off relation between sensitivity function and complementary sensitivity function. Consequently, this paper utilized the robust H_∞

*Corresponding author - e-mail: ngamroo@siit.tu.ac.th

control via normalized coprime factorization (NCF) approach^{6,7}. With the advent of NCF, the problem related to the selection of weighting function can be obviously simplified and also the robust H_∞ control design of the proposed PSS can be systematically explained.

This paper is organized as follows. The linearized model of a single machine connected to an infinite bus (SMIB) system is described in section 2. Section 3 presents the design procedure of H_∞ control via NCF approach. The numerical example of PSS design is given in section 4. Section 5 shows the performance of proposed PSS in comparison with a conventional PSS by simulation study. Finally, the conclusion is given in the last section.

2. SYSTEM MODEL

Figure 1 : System Configuration of SMIB.

A synchronous generator connected to an infinite bus through lossless transmission lines having an external reactance x_e is considered in this paper. A single line diagram of SMIB system is shown in Figure 1. The generator is fitted with an automatic voltage regulator (AVR), an excitation system, and the proposed PSS. A simple, but widely used linearized system equivalent to Figure 1 is the Heffron-Phillips model^{1,2}, which is shown in Figure 2. The SMIB system can be modeled as a forth-order system with changes in the load angle $\Delta\delta$, the rotor speed $\Delta\omega$, the internal voltage of the generator $\Delta e'_q$, and the field voltage ΔE_{fd} , as the state variables. The details of the modeling and system data are discussed more completely in appendix A. The initial conditions used as the design condition of the proposed PSS are $P_e = 0.8$ p.u., $Q_e = 0.4$ p.u., $x_e = 0.2$ p.u. The state equation of system in Figure 2 can be expressed as

$$\Delta X = A\Delta X + B\Delta u_{PSS} \quad (1)$$

$$\Delta Y = C\Delta X + D\Delta u_{PSS} \quad (2)$$

$$\Delta u_{PSS} = K(s)\Delta\omega \quad (3)$$

where the state vector $\Delta X = [\Delta\delta \ \Delta\omega \ \Delta e'_q \ \Delta E_{fd}]^T$ and the output vector $\Delta Y = [\Delta\omega]$. Δu_{PSS} is the control output signal of the proposed PSS ($K(s)$), which uses only the angular velocity deviation ($\Delta\omega$) as a feedback input signal. The details of system (1) and (2) are given in an appendix I. Note that the system (1) is a single input single output (SISO) system. Here, the H_∞ control via NCF approach is applied to design a robust PSS $K(s)$ for system (1) that is referred to as the nominal plant G .

Figure 2 : Linearized Model of SMIB System with the proposed PSS.

3. H_∞ CONTROL VIA NCF APPROACH

The design procedure of H_∞ control via NCF approach^{6,7} is divided into 4 steps as follows.

Figure 3 : Shaped Plant G_s and Robust Controller K .

Step 1. Loop-shaping

As shown in Figure 3, a precompensator, W_1 , and a postcompensator, W_2 , are employed to form the augmented plant $G_s = W_2 G W_1$, which is enclosed by a solid line. The designed robust controller $K = W_1 K_\infty W_2$ is enclosed by a dotted line where K_∞ is the H_∞ controller. The weighting function can be selected as $W_1 = W$ and $W_2 = 1$, since, the nominal plant G in system (1) is a SISO system. The details of selection of weighting function are later described in Step 3.

Step 2. H_∞ Robust Stabilization Problem Formulation

Figure 4 : H_∞ Robust Stabilization Problem.

The robust design objective is to stabilize the perturbed plant model G_Δ using a feedback controller K as shown in Figure 4. A robust stabilization problem is considered by using the normalized left coprime factorization to represent the nominal plant model G . Let the nominal plant model have a normalized left coprime factorization N_s, M_s such that $G_s = M_s^{-1} N_s$. Then a perturbed plant model G_Δ can be written as

$$G_\Delta = \{(M_s + \Delta M_s)^{-1} (N_s + \Delta N_s) : \|\Delta N_s \Delta M_s\| \leq 1/\gamma\} \quad (4)$$

where ΔM_s and ΔN_s are stable unknown transfer functions which represent system uncertainty in the nominal plant model. In (4), the minimum achievable value of γ , i.e. $\gamma_{\min}$, while retaining stability is implied to the maximum stability margin for the problem. Hence, γ is a limitation on the size of the perturbation that can exist without destabilizing the closed-loop system of Figure 4. With an advantage of selecting the NCF, $\gamma_{\min}$ can be explicitly obtained from (5), and no iteration is required as in the standard H_∞ control design^{6,7}.

$$\gamma_{\min} = \sqrt{1 + \lambda_{\max}(XZ)} \quad (5)$$

where $\lambda_{\max}(XZ)$ denotes the maximum eigenvalue of XZ . For a minimal state-space realization (A, B, C, D) of G_s , the value of X and Z are the unique positive solutions to the generalized control algebraic Riccati equation (GCARE)

$$(A - BS^{-1} D^T C)^T X + X(A - BS^{-1} D^T C) - XBS^{-1} B^T X + C^T R^{-1} C = 0 \quad (6)$$

and the generalized filtering algebraic Riccati equation (GFARE)

$$(A - BS^{-1} D^T C) Z + Z(A - BS^{-1} D^T C)^T - ZC^T R^{-1} CZ + BS^{-1} B^T = 0 \quad (7)$$

where $R = I + DD^T$, and $S = I + D^T D$. Consequently, to guarantee the stability of the nominal plant, the value of γ must be selected to be greater than $\gamma_{\min}^{6,7}$.

Step 3. Selection of weighting function

Based on the theory given in appendix B, the weighting function W is designed to reduce the peak value of the closed loop system to be less than $M_{p(new)}$ dB in the region of oscillation frequency (about 1 Hz). Hence, from (B2), the ∞ norm of the closed loop system can be expressed as

$$\|(I - GK)^{-1}G\|_{\infty} \leq \gamma / \|W\|_{\infty} = M_{p(new)} \quad (8)$$

As a result, the weighting function W is selected to satisfy (9) as the oscillation mode is in a region of low frequency^{6,7}.

$$|W|_{dB} \geq 20 \log \frac{\gamma}{10^{M_{p(new)}/20}} \quad (9)$$

After the weighting function W is selected, the shaped plant G_s is then established. Note that, the remaining conditions (B1), (B3), and (B4) are not investigated in this step.

Step 4. Determine the robust stabilization controller

The H_{∞} controller (K_{∞}) shown in Figure 3, which is the optimal controller for the selected $\gamma > \gamma_{\min}$, can be determined by

$$K_{\infty} = \begin{bmatrix} A + BF + \gamma^2 (L^T)^{-1} ZC^T (C + DF) & \gamma^2 (L^T)^{-1} ZC^T \\ B^T X & -D^T \end{bmatrix} \quad (10)$$

where $F = -S^{-1}(D^T C + B^T X)$ and $L = (I - \gamma^2)I + XZ$. Then the designed robust stabilization controller $K = WK_{\infty}$ is readily constructed. Subsequently, check whether or not the remaining conditions (B1), (B3), and (B4) are satisfied. If not, adjust the value of γ , then select a weighting function W and repeat the design procedures.

4. CONTROLLER DESIGN

In this section, an example of designing the robust H_∞ PSS is presented. Figure 5 shows a selected weighting function. Figures 6 to 9 show the stability conditions (B1), (B2), (B3), and (B4), respectively. As illustrated in Figure 7, the peak value of the open loop system, which is referred to as M_p is about -13 dB. In this example, it is approximately specified that the peak value of the closed loop system should be suppressed by the robust controller K until it is less than $M_{p(new)} = -30$ dB. By (5), $\gamma_{\min} = 2.64$ is obviously calculated. For the selection of γ , a small $\gamma \geq \gamma_{\min}$ (typically $2 < \gamma < 10$ in practice) guarantees that the final designed controller has a good robust stability property. However, if the selected value of γ is too small, conditions (B1)-(B4) may not be satisfied⁶. Here, γ is suitably set to 6.5. Then by (9), the lowest gain of weighting function W should be greater than 46.26 dB in the region of oscillation frequency. As shown in Figure 5, the weighting function is selected as $W = 700(s + 30)/(s + 100)$. Consequently, the shaped plant G_s can be established and the controller K_∞ can be constructed by (10). As a result, the sixth-order of robust controller K is obtained. Figures 6 to 9 confirm that the designed controller K satisfies conditions (B1)-(B4). These signify that both disturbance attenuation and robustness of K are guaranteed. Since, the coefficients of the 5th-6th orders of K are very small and can be neglected. Hence, the robust controller K can be approximated as

$$K(s) = 100 \frac{0.0005s^4 + 0.0218s^3 + 0.3122s^2 + 1.3395s + 3.0202}{0.0003s^4 + 0.0147s^3 + 0.428s^2 + 5.282s + 0.6874} \quad (11)$$

In addition, Figure 7 also indicates that the peak value of closed loop system $\|(I - GK)^{-1}G\|_\infty$ is less than -30 dB as a design specification.

Figure 5 : Weighting function W .

Figure 6 : (B1): $\|1/(I - GK)\|_\infty \leq \gamma$.

Figure 7: G and (B2): $\|G/(I-GK)\|_{\infty} \leq \gamma/\|W\|_{\infty}$. Figure 8: (B3): $\|K/(I-GK)\|_{\infty} \leq \gamma/\|W\|_{\infty}$.

Figure 9: (B4): $\|GK/(I-GK)\|_{\infty} \leq \gamma$.

5. PERFORMANCE EVALUATIONS AND RESULTS

Computer simulations are performed to evaluate the performance of the proposed PSS. The designed robust PSS is compared with the conventional lead-lag controller PSS

(CPSS) $K_c(s) = K_s \frac{sT_w}{(1+sT_w)} \frac{(1+sT_1)^2}{(1+sT_2)^2}$ obtained from reference 5. All parameters are

adjusted and set at $K_s = 5.5$, $T_w = 2.0$ s, $T_1 = 0.1732$ s, and $T_2 = 0.0577$ s. For both the proposed PSS and the CPSS, the limits on PSS output (Δu_{PSS}) are ± 0.05 p.u. and the limits on ΔE_{fd} are ± 6.0 p.u. Note that each PSS has the same control capacity. The system responses with PSSs are examined for various cases of disturbances. It is assumed that there is a small disturbance of 5% (0.05 p.u.) step response in ΔV_{ref} at $t = 0.0$ s. Figure 10 illustrates the dominant eigenvalues of operating conditions in Table 5.1. Figure 11 (a)-(d) shows system responses of the nominal closed-loop system of the

Table 1 : Operating Conditions.

System Parameters	(a) Normal Condition	(b) Weak Line Condition	(c) Heavy Load & Weak line	(d) Plant in Unstable State
P (p.u.)	0.8	0.8	<u>0.95</u>	0.95
Q (p.u.)	0.4	0.4	0.4	0.4
x_e (p.u.)	0.2	<u>0.8</u>	0.8	0.8
D (p.u.)	0.0	0.0	0.0	<u>-10.0</u>

uncontrolled plant as well as plants fitted with the CPSS and the proposed PSS, at four different operating conditions.

Operating condition (a) is the design point for CPSS and proposed PSS. As expected, both the CPSS and the proposed PSS can effectively damp the system oscillations. However, the proposed PSS has the smallest overshoot and first swing. Operating condition (b) is a case of a weakly connected system ($x_e = 0.8$ p.u.). This situation can cause low-frequency oscillations. As observed from Figure 11 (b), the performance of the CPSS deteriorates with the increase in transmission line impedance. Meanwhile, the proposed PSS provides better damping. Figure 11 (c) shows the system responses for heavy load and weak transmission line. As observed, the CPSS fail to stabilize the system at operating condition (c). For operating condition (d), it is assumed that the generator is operated under heavy load and weak transmission line condition. Furthermore, to take the effect of the uncertainty in the damping coefficient D into

Figure 10 : The root loci of eigenvalues.

(a) Normal Condition.

(b) Weak Line.

(c) Heavy Load and Weak Line.

(d) Unstable State Plant (negative damping).

Figure 11 : Responses of Electrical Power Deviation

account, it is assumed that the power system is in an unstable state since the value of D was reduced from 0.0 to -10.0 (negative damping). As shown in Figure 11 (d), the CPSS completely loses the damping effect as in case of no PSS. On the contrary, the proposed PSS is extremely robust against the heavy load situation and negative damping. The power oscillation is perfectly stabilized. These results clearly confirm the superior robustness of the proposed PSS beyond the CPSS.

6. CONCLUSIONS

The new robust design of PSS using H_∞ control via normalized coprime factorization (NCF) approach is proposed in this paper. The main outcomes can be summarised as follows:

- (i) With the advent of NCF, not only the design of H_∞ control can be systematically applied to the problem but also the selection of weighting function is significantly simplified.

- (ii) The proposed robust controller uses only a speed deviation of generator as the feedback input signal. More specifically, the practical realization in power systems can be easily implemented.
- (iii) Simulation results clearly ensure both the performance and robustness of the proposed robust PSS.

For the future development,

- (i) The optimisation techniques such as tabu search, genetic algorithm etc. will be applied to simplify the selection of weighting function in H_∞ control design.
- (ii) The proposed robust PSS will be applied to stabilization in multi-machine power system

7. REFERENCES

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APPENDIX A

The linearised system equations (1) and (2) can be expressed as

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{e}'_q \\ \Delta \dot{E}_{fd} \end{bmatrix} = \begin{bmatrix} 0 & \omega_B & 0 & 0 \\ -K_1 / M & -D / M & -K_2 / M & 0 \\ -K_4 / T'_{do} & 0 & -1 / T'_{do} & 1 / T'_{do} \\ -K_a K_5 / T_a & 0 & -K_a K_6 / T_a & -1 / T_a \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta e'_q \\ \Delta E_{fd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_a / T_a \end{bmatrix} \Delta u_{PSS} \quad (A1)$$

$$\Delta\omega = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta e'_q \\ \Delta E_{fd} \end{bmatrix} + [0]\Delta u_{PSS} \quad (A2)$$

The constants K_1 to K_6 are defined as

$$\begin{aligned} K_1 &= \frac{E_b e_{qo} \cos \delta_o}{(x_e + x_q)} + \frac{(x_q + x'_d)}{(x_e + x'_d)} E_b i_{qo} \sin \delta_o \\ K_2 &= \frac{E_b \sin \delta_o}{(x_e + x'_d)} \\ K_3 &= \frac{(x_e + x'_d)}{(x_d + x_e)} \\ K_4 &= \frac{(x_d + x'_d)}{(x_e + x'_d)} E_b \sin \delta_o \\ K_5 &= \frac{x_q v_{do} E_b \cos \delta_o}{(x_e + x_q) v_{to}} - \frac{x'_d v_{qo} E_b \sin \delta_o}{(x_e + x'_d) v_{to}} \\ K_6 &= \frac{x_e}{(x_e + x'_d)} \left(\frac{v_{qo}}{v_{to}} \right) \end{aligned} \quad (A3)$$

where

Δ	: small deviation	δ	: rotor angle
ω_B	: base speed	M	: inertia constant
D	: damping coefficient	ω	: rotor angular speed
x'_d	: d-axis transient reactance	x_d, x_q	: d and q axes synchronous reactance
x_e	: line reactance	i_d, i_q	: d and q axes generator currents
v_d, v_q	: d and q axes generator voltage	v_t	: generator terminal voltage
E_b	: infinite bus voltage	e'_q	: voltage proportional to field flux linkages
E_{fd}	: field voltage	K_a	: AVR gain
T_a	: AVR time constant	V_{ref}	: AVR reference input
P_e	: generator active power	Q_e	: generator reactive power
T'_{do}	: d-axis transient open circuit time constant		
subscript o : steady state value			

System Data

x_d	x'_d	x_q	T'_{do}	E_b	M	ω_B	K_a	T_a
2.0 p.u.	0.244 p.u.	1.91 p.u.	4.18 sec.	1.0	6.5 sec.	314.15 rad/sec	50.0	0.05 sec.

Appendix B

Theory The necessary and sufficient condition of robust controller K for stabilizing the perturbed plant G_Δ is

$$\left\| \begin{bmatrix} I \\ W^{-1} K \end{bmatrix} (I - GK)^{-1} [I \ G W] \right\|_\infty \leq \gamma \quad (\text{B0})$$

From (B0), four conditions for control design can be derived as

$$\|1 / (I - GK)\|_\infty \leq \gamma \quad (\text{B1})$$

$$\|G / (I - GK)\|_\infty \leq \gamma / \|W\|_\infty \quad (\text{B2})$$

$$\|K / (I - GK)\|_\infty \leq \gamma \|W\|_\infty \quad (\text{B3})$$

$$\|GK / (I - GK)\|_\infty \leq \gamma \quad (\text{B4})$$

The left sides of (B1) and (B2) are the ∞ norms of sensitivity function and closed loop system, respectively. Both conditions are related to the disturbance attenuation performance of controller. Conditions (B3) and (B4) show the robustness of controller. If the designed controller K satisfies with conditions (B1) - (B4), both performance and robustness of controller K are guaranteed. Note that, only condition (B2) is used for the selection of weighting function W in design step 3. The satisfactions of remaining conditions (B1), (B3) and (B4) are investigated later.